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The thermal regime of vapour bubble collapse at different Jacob numbers

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Abstract—For the case when heat transfer is the main process responsible for the collapse of bubbles, the numerical solution is obtained for a non-linear non-steady problem of heat and mass transfer between a spherical vapour bubble and liquid with external pressure increasing step-wise. The Jacob number is the only similarity criterion of this problem. A set of functions $R(t)$ (R is the bubble radius and t the time) is tabulated over a wide range of Jacob numbers ($0.01 < Ja < 1000$). It is shown that the variation of Ja entails a qualitative change in the form of the function $R(t)$. When $Ja = 0$, the function $R(t)$ is convex. When $0 < Ja < 2$, the curves $R(t)$ are S-shaped, and when $Ja > 2$ they are concave. Tabulated numerical results taken together with analytical formulae for the limiting cases $Ja \to 0$ and $Ja \to \infty$ constitute a simple method of calculating $R(t)$ for specific bubbles. An interpolation formula is obtained for determining the time of complete collapse of a bubble. The results of predictions are compared with experimental data.

1. INTRODUCTION

THE COLLAPSE of a single spherical vapour bubble after the jump in external pressure represents a complex process. An extensive study of this process implies regard for many physical factors [1]. Thus, numerical experiments presented in ref. [2] incorporated the effects of liquid inertia, heat transfer in both phases, kinetics of phase transition, liquid viscosity, and of the surface tension. The results of such calculations depend on a great number of dimensionless parameters associated with thermophysical properties, bubble sizes and pressure jumps, and each new case of collapse requires new computations based on a complex programme.

Another approach, which is used in the present work, consists in the study of specific cases, when it is possible to isolate one dominant factor essentially responsible for the process of collapse. Then the variation of one similarity criterion corresponding to this factor will exhaust the variety of possible situations. The results of the problem solution form a one-parameter set and can be tabulated. The tabulated results may be used to interpret experimental data and to verify approximate analytical methods. The need for new calculations is thus avoided.

The inertia of liquid and heat transfer between a bubble and liquid are the two main factors responsible for the collapse [1, 2]. Lord Rayleigh obtained [3] an analytical solution for the case when heat transfer occurred quickly and the process was of a purely hydrodynamic character, i.e. liquid motion to the bubble centre was controlled only by the liquid inertia. The inverse situation is considered in the present work. The collapse is assumed to occur rather slowly so that the pressure difference between the liquid and the bubble has time to equalize. In this case the process is controlled by heat transfer from the bubble to the liquid. Mathematically, this means that the heat conduction equation for the liquid outside the bubble needs to be solved taking into account the motion of the bubble boundary. It appears that the assignment of the sole similarity criterion—the Jacob number—is sufficient for an adequate dimensionless description of the process. The estimates given in ref. [1] make it possible to outline the boundaries of a purely thermal regime. Specifically, transition to thermal collapse always occurs with a decrease of the value of the pressure jump. Thus, for water boiling at 100°C the boundary of the thermal regime corresponds to pressure jumps of the order of 0.1 atm or smaller for the bubble size of about 1 mm.

The statement of the problem as well as numerical and analytical results for the time dependence of the bubble radius are given below. Taken together these results give the solution of the problem for the entire range of Ja from $-\infty$ to ∞. The numerical algorithm is described in the Appendix.

2. STATEMENT OF THE PROBLEM

The radius R_∞, pressure p_0 and temperature T_0, with $p_0 = p_S(T_0)$, correspond to the non-perturbed state of a bubble. After a sharp increase in the external pressure Δp, there occur the equalization of pressure and attainment of thermodynamic equilibrium in the bubble. The bubble becomes somewhat compressed [4]. In the model under consideration, these processes are rather rapid (instantaneous). From here on, the pressure and temperature in the bubble settle at the values $p_0+\Delta p$ and $T_0+\Delta T$, respectively, with $p_0+\Delta p = p_S(T_0+\Delta T)$. The temperature difference between the bubble and liquid is constant during the whole process because the liquid–vapour equilibrium

NOMENCLATURE

a	dimensionless radius of a bubble, R/R_0
c	heat capacity of liquid at constant pressure
erfc (x)	$(2/\sqrt{\pi})\int_x^\infty \exp\{-z^2\}\,dz$
h	enthalpy of vaporization
Ja	Jacob number, $c\rho_l\Delta T/h\rho_v$
R	radius of bubble
r	radial coordinate
s	$2\sqrt{\tau}$
T	temperature
t	time
u	$(r^3-R^3)/R_0^3$
v	a^3
x	$(r-R)/R_0$.

Greek symbols

α	thermal diffusivity of liquid
θ	dimensionless temperature, $(T-T_0)/\Delta T$
κ	'quasi-self-similarity' variable, x/s
ρ_v	density of vapour
ρ_l	density of liquid
τ	dimensionless time, $\alpha t R_0^{-2}$
τ^*	another dimensionless time, $4\pi^{-1} Ja^2\tau$.

Subscripts

0	initial state of bubble
c	instant of complete collapse ($a = 0$)
s	saturation state.

conditions must be satisfied. The corresponding heat flux produces vapour condensation on the walls of the bubble, in consequence of which the bubble decreases till its complete collapse. This process is described by [4, 5]

$$\frac{\partial T}{\partial t}+\frac{dR}{dt}\frac{R^2}{r^2}\frac{\partial T}{\partial r}=\alpha\left(\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}\right) \quad (1)$$

$$\frac{dR}{dt}=\frac{c\rho_l\alpha}{h\rho_v}\left.\frac{\partial T}{\partial r}\right|_{r=R};\quad R(0)=R_0 \quad (2)$$

$$T(r,t)=T_0+\Delta T;\quad T(r,0)=T(\infty,t)=T_0. \quad (3)$$

Here, $T(r, t)$ is the spherically symmetric temperature field in the liquid surrounding the bubble, $r > R$. Equation (1) describes a change in $T(r, t)$ due to heat conduction (the right-hand side of equation (1)) and convective radial heat transfer (the term $(dR/dt)(R^2/r^2)(\partial T/\partial r)$). Equation (2) determines the velocity of the bubble boundary due to the heat flux from bubble to liquid and vapour condensation. In equation (2), small terms of the order of $\rho_v/\rho_l \ll 1$ have been omitted. The system of equations (1)–(3) is nonlinear in $T(r, t)$, and this explains its complexity for analytical solution.

It should be emphasized that the quantity R_0 corresponds to the start of thermal collapse and that $R_0 < R_\infty$. In ref. [4], a relation between R_0 and R_∞ was derived on the basis of thermodynamic considerations. It can be assumed for estimation that $(R_\infty - R_0)/R_\infty \approx \Delta p/3p_0$ when $\Delta p < p_0$.

Now, turn to the dimensionless variables θ, x, τ and a instead of T, r, t and R. Then, the system of equations (1)–(3) will take the form

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial x^2}+\frac{\partial\theta}{\partial x}\left\{\frac{da}{d\tau}\left[1-\left(\frac{a}{x+a}\right)^2\right]+\frac{2}{x+a}\right\} \quad (4)$$

$$\frac{da}{d\tau}=Ja\left.\frac{\partial\theta}{\partial x}\right|_{x=0};\quad a(0)=1 \quad (5)$$

$$\theta(0,\tau)=1;\quad \theta(x,0)=\theta(\infty,\tau)=0. \quad (6)$$

It is seen that the solution of the system of equations (4)–(6) depends only on the value of Ja. The Jacob number has the following physical meaning. It will be assumed that the function $T(r)$ has the shape of a step, i.e. the liquid layer adjacent to the bubble is heated to the temperature $T_0+\Delta T$, whereas the rest liquid has the initial temperature T_0. Then, it is easy to show that Ja is equal to the ratio of the bubble volume in the unperturbed state $(4/3\pi R_0^3)$ to the liquid volume heated to the end of collapse. From what has been said above, the reason can be understood for a strong qualitative dependence of the character of the problem solution on the value of Ja. In fact, when $Ja \gg 1$, heat is transferred only in a comparatively thin boundary layer of liquid around the bubble. Conversely, at small values of Ja a great liquid volume that is much larger than the bubble volume is heated. Obviously, in these two cases the heat transfer follows different patterns.

3. RESULTS OF NUMERICAL SOLUTION

The values of the function $\tau^*(a)$ obtained at different values Ja are presented in Table 1. This very material is also given graphically in Figs. 1–3. It is seen that the function $\tau^*(a, Ja)$ increases monotonically with Ja. When $Ja > 2$, the curves $a(\tau^*)$ have a concave shape. When $Ja \to \infty$, they approach the limiting function designated by a dashed line in Fig. 1. At high Ja it is possible to distinguish the initial stage of collapse, over which all of the curves virtually merge, and the final stage, over which the functions $a(\tau^*)$ are close to linear and their slope depends strongly on Ja. The final stage, which accounts for the greater portion of the total time of collapse, corresponds to a relatively small range of the values of a.

When $Ja < 2$, the curves $a(\tau^*)$ have a distinct S-shape (Figs. 2 and 3). A convex portion increasing with a decrease of Ja appears on the curves. The limiting curves at $Ja = 0$ (the dashed-dotted lines) are purely convex. They have a vertical tangent line at the

Table 1. The values of the function $\tau^*(a, Ja)$ obtained by numerical solution of the system of equations (1)–(3). The values of $\tau^*(a, \infty)$ are calculated from formula (12)

a/Ja	0.01	0.02	0.1	0.2	0.5	1.0	2.0	5.0
0.9	0.000877	0.00153	0.00435	0.00602	0.00803	0.00915	0.00987	0.0104
0.8	0.001853	0.00339	0.01137	0.01740	0.0268	0.0335	0.0388	0.0431
0.7	0.002762	0.00515	0.01880	0.03043	0.0513	0.0691	0.0854	0.1007
0.6	0.003576	0.00676	0.0260	0.0437	0.0788	0.1126	0.1487	0.1863
0.5	0.00428	0.00817	0.0327	0.0564	0.1070	0.1609	0.2234	0.3025
0.4	0.00488	0.00936	0.0385	0.0680	0.1345	0.2111	0.309	0.452
0.3	0.00535	0.01032	0.0435	0.0781	0.1597	0.2603	0.400	0.634
0.2	0.00570	0.01104	0.0473	0.0862	0.1813	0.305	0.491	0.845
0.1	0.00592	0.0115	0.0499	0.0918	0.1975	0.341	0.571	1.067
0.05	0.00598	0.0116	0.0507	0.0936	0.203	0.354	0.602	1.167
0.0	0.00601	0.0117	0.0511	0.0945	0.206	0.361	0.620	1.23
0.9	0.0105	0.0107	0.0107	0.0107	0.0107	0.0107	0.0107	0.0107
0.8	0.0448	0.0457	0.0463	0.0465	0.0466	0.0466	0.0466	0.04667
0.7	0.1076	0.1115	0.1140	0.1148	0.1153	0.1155	0.1156	0.1156
0.6	0.2056	0.2174	0.2254	0.2282	0.2296	0.2305	0.2308	0.2311
0.5	0.348	0.378	0.400	0.408	0.412	0.415	0.416	0.4167
0.4	0.548	0.619	0.675	0.696	0.708	0.715	0.718	0.7200
0.3	0.820	0.979	1.121	1.182	1.216	1.237	1.245	1.252
0.2	1.180	1.526	1.906	2.097	2.212	2.291	2.318	2.347
0.1	1.630	2.358	3.46	4.23	4.81	5.28	5.47	5.67
0.05	1.86	2.89	4.81	6.58	8.37	10.25	11.17	12.33
0.0	2.04	3.35	6.38	10.3	16.7	31.1	49.7	∞

end of collapse and a finite slope at the start of collapse. The curves presented in Figs. 2 and 3 illustrate the character of the approximation to the limiting curves. With any $Ja > 0$ the initial stage of collapse always has a vertical tangent and, consequently, is concave. However, the extent of this stage decreases when $Ja \to 0$. Analytical study of the system of equations (4)–(6) for $Ja \to 0$ and $Ja \to \infty$, which is presented in the following sections, makes it possible to obtain the limiting curves in explicit form and to understand the above-described behaviour of the set of $a(\tau^*, Ja)$ curves.

4. THE LIMITING CASE $Ja \to \infty$

Let another substitution of variables be made: the variables a and x be replaced by v and u. Moreover, the derivative $\partial\theta/\partial\tau$ will be replaced by $\partial\theta/\partial v$. The 'time' v varies from 1 to 0. After identical transformation the following equation will be obtained for determining the temperature field $\theta(u, v)$:

$$\frac{\partial\theta}{\partial v}\frac{\partial\theta}{\partial u}\bigg|_{u=0} = Ja^{-1}\frac{\partial}{\partial u}\left[\left(1+\frac{u}{v}\right)^{4/3}\frac{\partial\theta}{\partial u}\right] \quad (7)$$

$$\theta(0, v) = 1; \quad \theta(\infty, v) = \theta(u, 1) = 0. \quad (8)$$

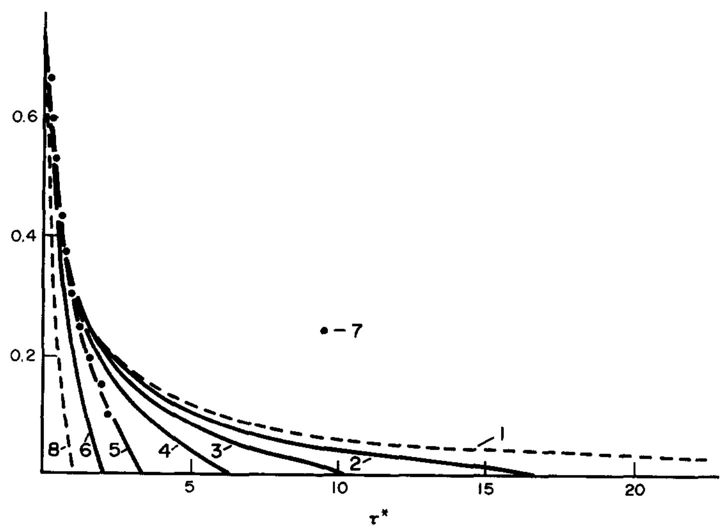

FIG. 1. The set of curves of time vs bubble collapse radius, $a(\tau^*, Ja)$. Solid lines, results of numerical calculations: 1, $Ja = \infty$, formula (12); 2, $Ja = 200$; 3, $Ja = 100$; 4, $Ja = 50$; 5, $Ja = 20$; 6, $Ja = 10$; 7, experimental data [6]; 8, $a = 1 - \sqrt{\tau^*}$ (see Section 8).

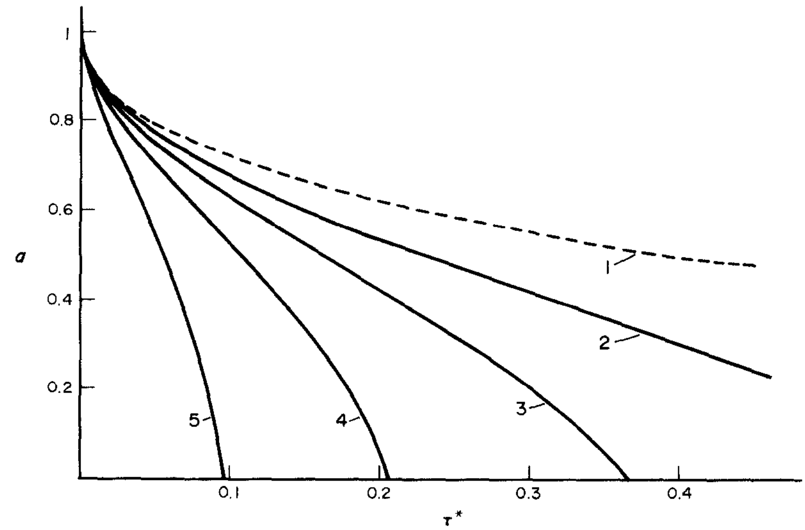

FIG. 2. The set of functions $a(\tau^*, Ja)$ for smaller values of Ja than those given in Fig. 1 : 1, formula (12); 2, $Ja = 2$; 3, $Ja = 1$; 4, $Ja = 0.5$; 5, $Ja = 0.2$.

This transformation of the initial system enables one to discard the convective term, but the form of the Laplacian becomes more complicated. Consider the conditions under which the right-hand side of equation (7) can be simplified by regarding $u \ll v$, i.e. assuming that the heated liquid layer is thin as compared with the bubble size. Obviously, this is always the case at the start of collapse. However, when $Ja \gg 1$, the range of validity of the thin heated layer approximation is much wider. Indeed, assume that most of the vapour has been already condensed. Then, according to the interpretation given above of the physical sense of the Jacob number, one can obtain: $\Delta a \cdot a^2 \sim Ja^{-1}$, where Δa is the characteristic thickness of the heated layer. Consequently, the condition for the heated layer to be thin ($\Delta a \ll a$) is equivalent to

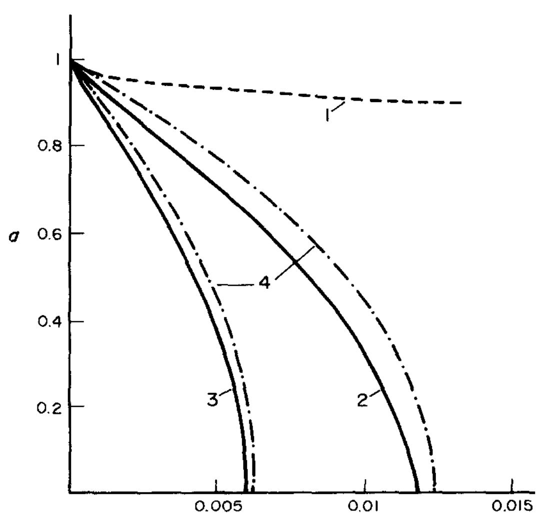

FIG. 3. The set of functions $a(\tau^*, Ja)$ for $Ja \ll 1$: 1, formula (12); 2, $Ja = 0.02$; 3, $Ja = 0.01$; 4, same curves according to formulas (14) and (16).

the inequality $Ja \cdot a^3 \gg 1$. This inequality is satisfied at any $a > 0$, when $Ja \to \infty$. Therefore, the limiting solution, which corresponds to $Ja \to \infty$, can be obtained from the following equation derived from equation (7) by eliminating the term u/v which is small as compared with unity:

$$\frac{\partial \theta}{\partial v}\frac{\partial \theta}{\partial u}\bigg|_{u=0} = Ja^{-1}\frac{\partial^2 \theta}{\partial u^2}. \tag{9}$$

Equation (9) has an exact solution

$$\theta(u, v)\,\mathrm{erfc}\,\{Ja \cdot u[\sqrt{\pi}(1-v)]\}. \tag{10}$$

Equation (5) in the new variables can be integrated in explicit form

$$\tau^* = \int_1^v \mathrm{d}v \frac{4Ja}{9\pi} v^{-4/3} \bigg/ \frac{\partial \theta}{\partial u}\bigg|_{u=0} \tag{11}$$

The substitution of equation (10) into integral (11) yields the well-known formula first obtained in ref. [6]

$$\tau^* = \tfrac{2}{3}a^{-1} + \tfrac{1}{3}a^2 - 1. \tag{12}$$

Florschuetz and Chao [6] used the thin heated layer method of Plesset and Zwick [7]. The above derivation of equation (12) is distinguished by employing the quantity $v = (R/R_0)^3$ rather than the variable

$$\int_0^t R^4\,\mathrm{d}t$$

which is used in the work of Plesset and Zwick [7]. The use of the variable v makes it possible to obtain the final result—formula (12)—much more easily than in ref. [6]. It should be emphasized however that the present version of the thin heated layer method does not lead to new solutions. The method of Plesset and Zwick gives the following solution of the problem under consideration:

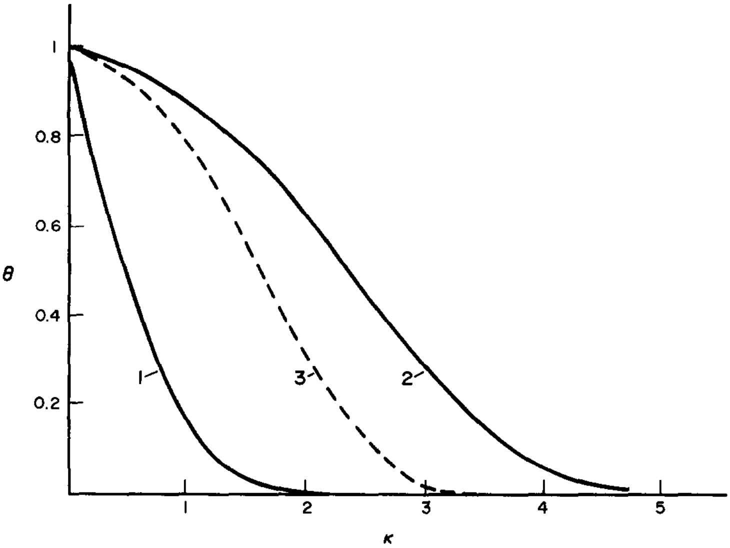

FIG. 4. Temperature profiles at the start and end of collapse for $Ja = 37.5$ (κ is the 'quasi-self-similarity' variable); 1, $\tau = 0$, $a = 1$; 2, $\tau = \tau_c$, $a = 0$; 3, $a = 0.1$. 1 and 2, numerical calculations; 3, formula (10).

$$\theta(r,t) = \operatorname{erfc}\left[(r^3 - R^3)\Big/6\left(\alpha\int_0^t R^4\,\mathrm{d}t\right)^{1/2}\right]. \quad (10')$$

Despite the difference in appearance, solutions, equations (10) and (10′), are identical. This can be easily proved by taking into account the relation between R and t expressed as function (12). However, equation (10) is simpler; it directly relates the temperature profile to the bubble radius and does not involve the time. Moreover, the form of equation (10) vividly displays the role of Ja: as Ja increases, the heated layer becomes thinner.

When $Ja \gg 1$, function (10) is close to an exact solution, and gives the shape of the temperature profile close to the real one, as can be seen from Fig. 4. More than likely this is the reason for a good agreement between formula (12) and the results of numerical calculations. This agreement also persists beyond the region of validity of the original approximation $\Delta a \ll a$. For example, when $Ja = 1000$ and $a = 0.1$, $Ja \cdot a^3 = 1$ and the layer of heated liquid is not thin. However, the value of τ^* calculated by equation (12) differs by only 4% from the results of numerical calculations (Table 1). Thus, equation (12) not only gives an exact limiting law for the bubble collapse, but also represents a very good approximation at large Ja.

5. THE LIMITING CASE $Ja \to 0$

When $Ja \to 0$, equation (5) yields that $\mathrm{d}a/\mathrm{d}\tau \to 0$. For such an 'infinitely slow' collapse the temperature profile has a quasi-stationary shape

$$T(r,t) = T_0 + \Delta T\frac{R(t)}{r}. \quad (13)$$

The form of the function $R(t)$ can be easily obtained by substituting function (13) into equation (5) and integrating

$$a = (1-\tau^*/\tau_c^*)^{1/2}. \quad (14)$$

Here, $\tau_c^* = 2Ja/\pi$. The method of successive approximations will now be used. To this end, it is necessary to substitute solution (13) into the left-hand side of equation (1) and to solve the following equation for $T^{(1)}$

$$\frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T^{(1)}}{\partial r}\right) = \frac{\partial T^{(0)}}{\partial t} + \frac{\mathrm{d}R^{(0)}}{\mathrm{d}t}\left(\frac{R^{(0)}}{r}\right)^2\frac{\partial T^{(0)}}{\partial r}. \quad (15)$$

Here $T^{(0)}$ and $R^{(0)}$ correspond to equations (13) and (14). Omitting the details of the solution of equation (15), the final result is obtained: equation (14) retains its form, but now

$$\tau_c^* = \frac{2Ja}{\pi(1+1.5Ja)}. \quad (16)$$

In Fig. 3, the limiting curves, equations (14) and (16), are compared with the results of numerical calculations. The main specific feature of the quasi-steady collapse—its acceleration to the end of the process—is very clearly seen.

6. COMPARISON WITH EXPERIMENTAL DATA

Unfortunately, there are no experimental data that can represent a change in the pattern of bubble collapse with variation of Ja within wide ranges. The majority of experiments were carried out with water at 100°C and at Δp of the order of a fraction of an atmosphere. In this case, $Ja \sim 20$–50. For such values of Ja, numerical calculations differ from equation (12) only when $a \lesssim 0.3$. Therefore, to compare with the predicted results (Fig. 1) the experimental data of ref. [6] were taken for a bubble with a remarkably low gas content (air concentration amounted only to

2×10^{-4}). Experimental points given in ref. [6] for $a > 0.1$ are close to curve 5 in Fig. 1 ($Ja = 20$). In fact, the predicted curve for this bubble ($Ja = 37.5$ [6]) should lie a little to the right, closer to curve 4 ($Ja = 50$). Thus, the experimental rate of bubble collapse is somewhat higher than the predicted one. This seems to be due to the bubble motion which enhances heat transfer.

7. TIME OF COMPLETE COLLAPSE OF A BUBBLE

According to equation (12), $\tau^* = \infty$ at $a = 0$. This indicates that $\tau_c^* \to \infty$ when $Ja \to \infty$. An attempt has been made to obtain additional information about τ_c^* by solving equation (7) for $Ja \gg 1$ by the method of successive approximations. This can be done by different techniques, but all of them are not mathematically quite rigorous, since at small values of a the values of τ^* differ appreciably from those given by equation (12). However, all these attempts lead to the formulas of the form $\tau_c^* = C Ja^{2/3}$ where $C \sim 1$. This prediction, which has the character of a plausible hypothesis, checks excellently with the results of numerical calculations at $Ja = 20$, 50, 100, 200, 500, 1000 (Fig. 5).

At small values of Ja, the function $\tau_c^*(Ja)$ is given by equation (16). All of these data are described by a single interpolation formula

$$\tau_c^* = 0.5Ja(0.69+\sqrt{Ja})^{-2/3}. \qquad (17)$$

Despite the simplicity of this relation, it gives an error not higher than 0.5% as compared with the results of numerical calculations. In Fig. 5 the curve, which corresponds to equation (17), is compared with experimental data of refs. [6, 8]. The experimental values of τ_c^* have been determined in the present work by extrapolating the experimental curves $a(\tau^*)$ to $a = 0$. This procedure is necessary because actual bubbles do not collapse to the end owing to the presence of dissolved gases in them. Figure 5 shows that equation (17) agrees well with the experimental data. However, for actual collapse equation (17) will always somewhat overestimate τ_c^* due to the disregard of the motion of bubbles which enhances heat transfer.

8. CONCLUDING REMARKS

The estimates of $a(\tau^*)$ and τ_c^* which can be obtained by the substitution of the temperature field

$$\theta(r, t) = \operatorname{erfc}[(r-R)/2(\alpha t)^{1/2}]$$

into equation (2) (i.e. as a result of the exact solution of a plane thermal problem) are well known [6, 8]. It can be easily checked that this yields $a = 1-\sqrt{\tau^*}$, $\tau_c^* = 1$. Figures 1–3 and 5 show that these estimates may differ appreciably, by many orders, from correct values. This disadvantage seems to be also typical for a more general approach based on the use of the Duhamel integral for describing a heat flux from the bubble to liquid in the thermal regime with the external pressure varying arbitrarily. Especially marked deviations of the real behaviour of bubbles from the plane boundary approximation is to be expected at small values of Ja ($Ja \lesssim 1$).

Formulations of the problems which have been revealed, but not solved, in the present work are given below.

(1) The rigorous analytical investigation of the asymptotic law for $\tau_c^*(Ja)$ when $Ja \to \infty$ is absent. It would be of interest to prove rigorously that $\tau_c^* \sim C\ Ja^{2/3}$ and to calculate the value of C (or to reject this hypothesis).

(2) There is no rigorous investigation of the behaviour of the function $R(t)$ resulting from the solution

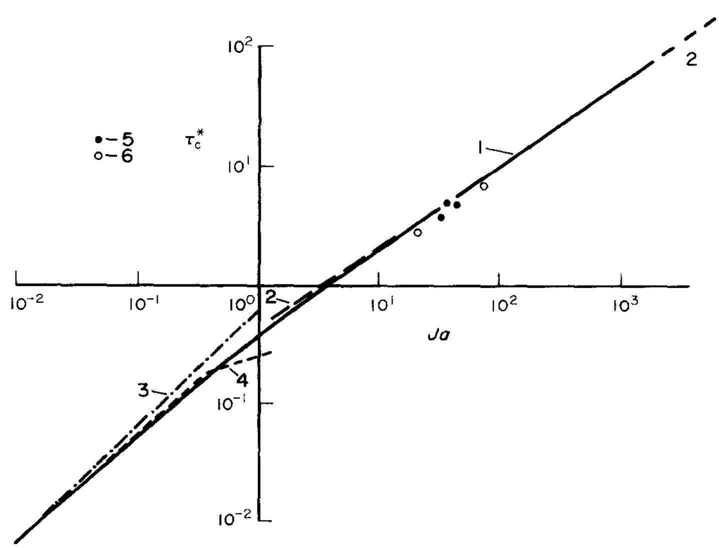

FIG. 5. Dependence of the dimensionless time of bubble collapse τ_c^* on Ja: 1, formula (17); 2, $\tau_c^* = 0.5Ja^{2/3}$; 3, $\tau_c^* = 2Ja/\pi$; 4, formula (16); 5, 6, experimental data of refs. [6, 8], respectively.

of the system of equations (1)–(3) at $R = 0$. Numerical calculations do not allow one to answer with certainty the simple question whether the rate of collapse dR/dt is finite or infinite at $R = 0$. There is also another question left untouched as to whether the curves $R(t)$ are purely concave when $Ja \gg 1$ or their convexity at the end of collapse becomes very small and indiscernible against the background of calculation errors. It is quite probable that the curves $R(t)$ are S-shaped for all Ja, but when $Ja \to \infty$, their convex portion flattens more and more, not losing yet its convexity.

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APPENDIX: ALGORITHM FOR NUMERICAL SOLUTION OF THE SYSTEM OF EQUATIONS (4)–(6)

The unknown function $\theta(x, \tau)$ is discontinuous at $\tau = x = 0$. Therefore, the straight numerical solution of the problem in these variables is impossible. The replacement of the variables x and τ by $\kappa = x/s$ and $s = 2\sqrt{\tau}$ allows the smoothing of the solution in both the time and space variables. The system of equations (4)–(6), transformed to new variables, acquires the form

$$\frac{\partial\theta}{\partial s} = \frac{1}{2s}\frac{\partial^2\theta}{\partial\kappa^2} + \frac{\partial\theta}{\partial\kappa}\left\{\frac{da}{ds}\left[1 - \left(\frac{a}{\kappa s + a}\right)^2\right] + \frac{\kappa}{s} + \frac{1}{\kappa s + a}\right\} \quad (A1)$$

$$\frac{da}{ds} = \frac{Ja}{2}\left.\frac{\partial\theta}{\partial\kappa}\right|_{\kappa=0}, \quad a(0) = 1 \quad (A2)$$

$$\theta(\kappa, 0) = \operatorname{erfc}(\kappa),\ \theta(0, s) = 1,\ \theta(\infty, s) = 0. \quad (A3)$$

The introduction of the variable $\kappa = x/s$ is reasonable because of its 'quasi-self-similarity' nature. By neglecting the motion of the bubble boundary and its curvature, it is possible to obtain the solution of equation (A1) in the form $\theta = \operatorname{erfc}(\kappa)$. For this reason the initial condition in the variables κ, s has the form of equation (A3). In the process of collapse the temperature field $\theta(\kappa, s)$ becomes noticeably deformed, but its extension along the κ-axis varies little (Fig. 4). For solving equation (A1), use was made of the doubly centred, four-point, always stable implicit difference scheme of the second-order approximation. Difference equations were solved by the factorization method. It should be noted that because of the presence of the terms $\sim s^{-1}$ on the right-hand side of equation (A1), this scheme has only the first-order approximation near $s = 0$. However, it is easy to prove that for any fixed $s > 0$ the second-order approximation is retained when the mesh width for κ is diminished. For calculating $\partial\theta/\partial\kappa|_{\kappa=0}$ and determining the function $a(s)$ by integrating equation (A2), expressions of the third-order approximation have been used, since only then the accuracy of calculation of $a(s)$ is not worse than that provided by the difference scheme used for the solution of equation (A1).

The accuracy of calculation was controlled with the aid of the thin heated layer approximation. To obtain this approximation, it is sufficient to neglect the term $(\kappa s + a)^{-1}$ within the braces of equation (A1) and also to expand the expression within the square brackets of equation (A1) into the series in the small parameter $x/a = \kappa s/a$

$$1 - \left(\frac{a}{\kappa s + a}\right)^2 \approx \frac{2\kappa s}{a}. \quad (A4)$$

In this approximation the solution of the system of equations (A1)–(A3) is given by formula (12). By substituting equation (A4) into equation (A1) and solving the resulting system numerically, the results must be obtained that coincide with those given by formula (12) within the errors of the numerical algorithm. Such a control algorithm is very convenient, because it differs from the main algorithm only by one Fortran operator and the control system of equations is very similar to the initial one. When performing calculations the results of which are listed in Table 1, the values of mesh widths for κ and s were of the order of 10^{-3}. The values of τ^* calculated for the thin heated layer approximation differed from those given by formula (12) only by the fifth decimal digit. Therefore, the three decimal digits retained in Table 1 can be regarded as reliable.

LE REGIME THERMIQUE DU COLLAPSUS DE BULLE DE VAPEUR A DIFFERENTS NOMBRES DE JACOB

Résumé—Dans le cas où le transfert thermique est le principal mécanisme responsable du collapsus des bulles, la solution numérique est obtenue pour un problème variable non linéaire de transfert de chaleur et de masse entre une bulle de vapeur sphérique et un liquide, avec une pression externe croissant par échelon. Le nombre de Jacob est le seul critère de similitude du problème. Un système de fonctions $R(t)$ (R est le rayon de bulle et t le temps) est tabulé pour un large domaine de nombre de Jacob ($0{,}01 < Ja < 1000$). On montre que la variation de Ja entraine un changement qualitatif de la forme de la fonction $R(t)$. Quand $Ja = 0$, la fonction $R(t)$ est convexe. Quand $0 < Ja < 2$, elle est concave. Des résultats numériques tabulés pris avec les formules analytiques pour les cas limites $Ja \to 0$ et $Ja \to \infty$ constituent une méthode simple de calcul de $R(t)$ pour des bulles spécifiques. Une formule d'interpolation est obtenue pour déterminer le temps de collapsus complet d'une bulle. Les résultats de prédiction sont comparés avec les données expérimentales.

DAS KOLLABIEREN VON DAMPFBLASEN BEI VERSCHIEDENEN JAKOB-ZAHLEN

Zusammenfassung—Für den Fall, daß die Wärmeübertragung der dominierende Vorgang beim Kollabieren von Blasen ist, wird eine numerische Lösung für das nichtlineare instationäre Wärme- und Stoffübergangsproblem zwischen einer kugelförmigen Dampfblase und der Flüssigkeit ermittelt, wobei der äußere Druck stufenweise gesteigert wird. Die Jakob-Zahl ist das einzige Ähnlichkeitskriterium für dieses Problem. Einige Funktionen $R(t)$ (Blasenradius R in Abhängigkeit von der Zeit t) werden über einen weiten Bereich der Jakob-Zahl ($0{,}01 < Ja < 1000$) tabelliert. Es wird gezeigt, daß eine Variation der Ja-Zahl eine qualitative Formänderung der Funktion $R(t)$ zur Folge hat. Bei $Ja = 0$ ist die Funktion $R(t)$ konvex, im Bereich $0 < Ja < 2$ besitzen die $R(t)$-Kurven S-förmige Gestalt, für $Ja > 2$ werden die Kurven konkav. Tabellierte numerische Ergebnisse zusammen mit analytischen Gleichungen für die Grenzfälle $Ja \to 0$ und $Ja \to \infty$ ergeben eine einfache Methode zur Berechnung von $R(t)$ für bestimmte Blasen. Die Zeit für das vollständige Kollabieren einer Blase wird aus einer Interpolationsformel ermittelt. Die Berechnungsergebnisse wurden mit experimentellen Daten verglichen.

ТЕПЛОВОЙ РЕЖИМ СХЛОПЫВАНИЯ ПАРОВОГО ПУЗЫРЯ ПРИ РАЗЛИЧНЫХ ЗНАЧЕНИЯХ ЧИСЛА ЯКОБА

Аннотация—Решена численно нелинейная нестационарная задача теплообмена парового пузыря с жидкостью при скачкообразном возрастании внешнего давления для случая, когда теплообмен является основным процессом, влияющим на схлопывание. Получено семейство зависимостей радиуса пузыря от времени $R(t)$ в широком интервале значений числа Якоба Ja—единственного критерия подобия данной задачи ($0{,}01 < Ja < 1000$). Показано, что при изменении Ja вид функции $R(t)$ качественно изменяется. При $Ja = 0$ функция выпукла, при $0 < Ja < 2$ кривая $R(t)$ имеет S-образную форму, при $Ja > 2$ она становится вогнутой. Табулированные результаты численного решения в совокупности с аналитическими формулами для предельных случаев $Ja \to 0$ и $Ja \to \infty$ составляют простой метод расчёта $R(t)$ для конкретных пузырей. Получена интерполяционная формула для определения времени полного смыкания пузыря. Результаты расчётов сопоставлены с экспериментальными данными.